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From the Editor-in-Chief

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In this column, edited by one of the occupants of the position of editor-in-chief, we relate comments from authors and readers concerning papers that have recently appeared in *Linear Algebra and its Applications*. The column will contain errata, additional references, and historical and other comments that we believe will be of interest to readers of the journal. With two volumes a year, each with 12 issues, we plan for this column to appear in the first issue of even-numbered volumes.

1. S.Furuichi, K.Kuriyama and K.Yanagi, Trace inequalities for products of matrices, 430 (2009), 2271–2276. In this paper, the following theorem is proved:
   For positive numbers $p_1, p_2, \cdots, p_m$ with $p_1 + p_2 + \cdots + p_m = 1$ and 2×2 positive definite matrices T and A, we have the inequalities
   $$Tr\left[\left(T^{1/m}A\right)^m\right] \le Tr\left[T^{p_1}AT^{p_2}A\cdots T^{p_m}A\right] \le Tr\left[TA^m\right].$$
   In the paper, it was remarked that if $m, n \ge 3$, the expression in the middle is not necessarily real. This leads to the following conjecture of the authors: Do the following inequalities hold or not, for positive numbers $p_1, p_2, \cdots, p_m$ with $p_1 + p_2 + \cdots + p_m = 1$ and $n \times n$ positive definite matrices T and A

   (i) $Tr\left[\left(T^{1/m}A\right)^m\right] \le Re\left\{Tr\left[T^{p_1}AT^{p_2}A\cdots T^{p_m}A\right]\right\}$.
   (ii) $\left|Tr\left[T^{p_1}AT^{p_2}A\cdots T^{p_m}A\right]\right| \le Tr\left[TA^m\right]$.

   Shigeru Furuichi and Minghua Lin have found a counter-example to this inequality as follows: Consider the two positive definite matrices A and B given in the paper: C.R. Johnson and C.J. Hilla, Eigenvalues of words in two positive definite letters, SIAM J. Matrix Anal. Appl., 23(2002), 916–928:
   $$A = \begin{pmatrix} 1 & 20 & 210 \\ 20 & 402 & 4240 \\ 210 & 4240 & 44903 \end{pmatrix}, B = \begin{pmatrix} 36501 & -3820 & 190 \\ -3820 & 401 & -20 \\ 190 & -20 & 1 \end{pmatrix}.$$
   For the above matrix B, put $T = B^3$, then (see p. 919 in the above paper)
   $$0 > Tr[ABA^2B^2] = Tr[B^2ABA^2] = Tr[T^{2/3}AT^{1/3}AT^0A].$$
   Thus we can take nonnegative p_1, p_2, p_3 satisfying $Tr[T^{p_1}AT^{p_2}AT^{p_3}A] < 0$ by continuity. Therefore (i) of the conjecture does not hold in general.

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In fact, for the above two positive matrices A and B,

$$Tr[B^{199/100}ABAB^{1/100}A] \simeq -2270.33,$$

by using Mathematica and MatLab.
However the following inequalities are still remain open.

$$Tr\left[\left(T^{1/m}A\right)^m\right] \le |Tr\left[T^{p_1}AT^{p_2}A\cdots T^{p_m}A\right]| \le Tr\left[TA^m\right],$$

for positive numbers $p_1, p_2, \cdots, p_m$ with $p_1 + p_2 + \cdots + p_m = 1$ and $T, A \in M_+(n, \mathbb{C})$, where $n \ge 3$ and $m \ge 3$.

2. J.C. Hou, C.K. Li and N.C. Wong, Maps preserving the spectrum of generalized Jordan product of operators, 432 (2010), 1049–1069. Jianlian Cui has pointed out to the authors that some arguments in the proof of Theorem 3.1 are not entirely clear and accurate, and the authors have provided the following corrections:

   - Remove the paragraph "In the case $s > r > 0, \ldots$. Thus $A^2 \neq 0$" after Claim 4, as we do not need this in the proof.
   - First line of the proof of Claim 6 in the proof of Theorem 3.1 should be "Let f be nonzero in X_1^*. Assume $\langle x_1, f\rangle = \langle x_2, f\rangle = 1$".
   - The proofs of Claim 6 and Case 2 of Claim 7 need some adjustment.
   - Lemma 3.8 should be changed to:
     Lemma 3.8 *Suppose* dim$X \ge 3$. *Let $P, Q \in \mathcal{I}_1(X)$. Then $PQ = 0 = QP$ if and only if there is $B \in \mathcal{B}(X)$, which can be chosen to have rank 2, such that $\sigma(PB + BP) = \{2, 0\}$, $\sigma(QB + QB) = \{-2, 0\}$, and $\sigma(BR + RB) = \{0\}$ whenever $R \in \mathcal{I}_1(X)$ satisfies $\sigma(PR + RP) = \sigma(QR + RQ) = \{0\}$.*

   One may see the details of these adjustments at
   http://arxiv.org/abs/1004.3832 (arXiv:1004.3832v2 [math.FA]), or at
   http://www.math.wm.edu/∼ckli/HLWaddendum.pdf.

3. Robert Shorten, Oliver Mason, and Christopher King, An alternative proof of the Barker, Berman, Plemmons (BBP) result on diagonal stability and extensions, 431(1) (2009) 34–40. The authors have provided the following clarification:
   In Lemma 3.1 of it is stated that the cone $\mathcal{C}_B$ defined in eqn. (7) is closed. However this cone, as defined, may fail to be closed. Since the proof of Lemma 3.1 requires all such cones to be closed, the correct statement is ($\overline{\mathcal{C}_B}$ denotes the closure of $\mathcal{C}_B$):

   Lemma 3.1' Let $A \in \mathbb{R}^{n\times n}$ be Hurwitz and let $M_i \in \mathbb{R}^{n\times n}$, for $i = 1, .., k$. Then there exists a positive definite P satisfying

$$A^TP + PA < 0, \quad M_i^TP + PM_i \le 0 \quad i = 1, \ldots, k \tag{1}$$

   if and only if there do not exist matrices X, Z with $X \ge 0, X \neq 0$ and $Z \in \overline{\mathcal{C}_B}$ such that

$$AX + XA^T + Z = 0. \tag{2}$$

   All results are unchanged. Proofs are as before with cones replaced by their closures where necessary. The proof of Lemma 3.1' is identical to the published proof of Lemma 3.1 except that (iii) is replaced by "There exist no matrices X, Z with $X \ge 0, X \neq 0$ and $Z \in \overline{\mathcal{C}_B}$ satisfying $AX + XA^T + Z = 0$," (iv) is replaced by "The two pointed convex cones $\mathcal{C}_A$ and $\overline{\mathcal{C}_B}$ intersect only at the origin", and $\mathcal{C}_B$ is replaced everywhere by $\overline{\mathcal{C}_B}$. In the sufficiency part of the proof of Theorem 3.1, eqn. (9) should be replaced by the equation $AX + XA^T + Z = 0$ where Z is in the closure of $\{\sum_{i=1}^{n-1}(B_iY_i + Y_iB_i) \ : \ Y_i = Y_i^T \ge 0\}$, and in subsequent statements $B_iY_i + Y_iB_i$ should be replaced by Z (note that the diagonal entries of Z must be non-positive, rather than strictly negative). Finally, in the proof of Theorem 4.1 the reference to positive semi-definite matrices $Y_1, \ldots, Y_n$ should be replaced by reference to Z in the closure of $\{\sum_{i=1}^{n-1}(B_iY_i + Y_iB_i) \; : \; Y_i = Y_i^T \geq 0\}$. An updated manuscript is available online at www.hamilton.ie.

4. G. Corach and A. Maestripieri, Polar decompositions of oblique projections, 433 (2010), 511-519. The authors have noticed that an older paper by I. Vidav contains some results similar to theirs. In particular, Theorem 2 of the paper "On idempotent operators in a Hilbert space", Publ. Inst. Math. (Beograd) 4(18) (1964), 157-163, by Ivan Vidav, solves the same problem treated in Theorem 6.3. Essentially, he proves that, given closed range operators A, B acting on a Hilbert space, which are positive semidefinite, then there exists an idempotent Q such that $QQ^* = A$ and $Q^*Q = B$ if and only if $ABA = A^2$ and $BAB = B^2$. In Theorem 6.3, they found the equivalent condition $P_{R(A)}BP_{R(A)} = P_{R(A)}$ and $P_{R(B)}AP_{R(B)} = P_{R(B)}$. In fact, one passes from Vidav's identities to theirs by multiplying at left and right by the Moore-Penrose inverse of A or B, respectively, and from theirs to Vidav's by multiplying by A or B, respectively. The identities of Vidav have the additional advantage that they are polynomial identities, and they have been studied in a much more general context than Hilbert space operators, for example, C. Schmoeger, Common spectral properties of linear operators A and B such that $ABA = A^2$ and $BAB = B^2$, Publ. Inst. Math. (Beograd) 79(93)(2006), 109-114.
5. Suk-Geun Hwang and Sung-Soo Pyo, The inverse eigenvalue problem for symmetric doubly stochastic matrices, 379 (2004), 77–83, and Maozhong Fang, A note on the inverse eigenvalue problem for symmetric doubly stochastic matrices, 432 (2010), 2925–2927. Fang gives a 3×3 counterexample to Proposition 1 of the first paper. Carlos Fonseca has sent a 2×2 counterexample. The 2-tuple $(1, -1)$ satisfies the hypothesis of Proposition 1 but the only doubly stochastic matrix with eigenvalues $1, -1$ is the backward identity matrix and it is not positive. Fonseca also pointed out that Theorem 4 in the paper by Hwang and Pyo appeared previously as Theorem 8 in the reference "Spectral properties of doubly-stochastic matrices" by H. Perfect and L. Mirsky (Monatsh. Math. 69 (1965), 35-57).
6. R. Shorten and K.S. Narendra, On a theorem of Redheffer concerning diagonal stability, 431 (2009), 2317–2329. Robert Shorten has provided the following corrections: In Lemma 4.2 the assumption that c^T is of the form $[\alpha, 0, 0, \ldots, 0]$ has been omitted. Namely, the lemma should read: (for clarification, by leading submatrix we mean the $n-1 \times n-1$ matrix obtained by removing the first row and column of Q).

   Lemma 4.2. Let A be a Hurwitz Metzler matrix. Let $B = A - bc^T$ be a Hurwitz matrix that is not necessarily Metzler, where $c^T = [\alpha, 0, 0, \ldots, 0]$, for some non-zero α . Let there exist a strictly positive diagonal matrix D such that $A^TD + DA = -Q_1 \leq 0$ with Q_1 singular and irreducible, and that the leading principal submatrix of Q_1 is positive definite. Suppose further that $B^TD + DB = -Q_2 \leq 0$ and that there is no other diagonal $D > 0$ satisfying the strict inequalities ($Q_1 < 0, Q_2 < 0$). Then, there exists a diagonal matrix Γ,whose diagonal entries are not all zero, such that $\det\Big(A + \Gamma B \Gamma) = 0$.

   Also, in section 4, the text "transcendal part is maximised" should read "transcendental part is minimized".
7. V. Nikiforov, The energy of C_4-free graphs of bounded degree, 428 (2008), 2569–2573. Xueliang Li and Jianxi Liu have observed that the two conjectures in this paper have been proved in a general form by C. Heuberger and S. Wagner who apparently were unaware of these conjectures: Chemical trees minimizing energy and Hosoya index, J. Math. Chem., 46(1) (2009), 214–230.
8. R.A. Brualdi, Spectra of digraphs, 432 (2010), 2181–2213. As stated in the paper, the inequality (24) in Theorem 8.3 was first obtained by S. Kirkland. The block matrix statement in Theorem 8.3 was first proved by S.V. Savchenko in reference [96]. Savchenko has also remarked that the results announced in reference [99] were proved in: S.P. Strunkov, On weakly cospectral graphs (Russian), Mat. Zametki 75 (4) (2004), 614–623; English trans. in Math. Notes 75 (4) (2004), 574–582. He also notes that according to this paper by S.P. Strunkov, two digraphs are weakly cospectral if and only if they have the same set of distinct eigenvalues. So, a priori, they can

be isomorphic to each other. The paper: S.P. Strunkov, On weakly cospectral graphs II (Russian), Mat. Zametki 80 (4) (2006), 627–629; English trans. in Math. Notes 80 (4) (2006), 590–592 is also devoted to this subject.

9. Z. Stanić, On nested split graphs whose second largest eigenvalue is less than 1, 430 (2009), 2200–2211. The author has provided the following correction. Page 2204, lines 22 and 23: The part “Hence, $a_3 \geq 3$ implies $a_1 + a_2 \leq 3$. Moreover, if $(a_1, a_2) \in \{(1, 2), (2, 1)\}$ then the determinant is less than zero for any choice of the remaining parameters." should be replaced with: “Hence, $a_3 \geq 3$ implies that at least one of parameters a_1, a_2 is equal to 1. Moreover, if $(a_1, a_2) \in \{(1, 2), (2, 1)\}$ then the determinant is less than zero for any choice of the remaining parameters. By putting $a_1 = 1$ and $a_2, a_3 \geq 3$ we get no solutions, while by putting $a_2 = 1$ we get the following solutions: $(a_1, a_2, a_3, a_4) = (3, 1, 6, l)$, $(4, 1, 4, l)$, $(6, 1, 3, l)$, $(7, 1, 3, 8)$, $(8, 1, 3, 4)$, $(9, 1, 3, 3)$ and $(13, 1, 3, 2)$, where $l \geq 2$."
Thus, Theorem 4.1 should read:

Theorem 4.1. Let $G = C(a_1, a_2, \ldots, a_n)$ be a connected NSG satisfying $\lambda_2(G) < 1$. If $n \leq 4$ then G is an induced subgraph of some of the following graphs:

$C(a_1, a_2, a_3)$, where $(a_1, a_2, a_3) = (k, m, 2)$, $(3, 1, 7)$, $(5, 1, 5)$, $(k, 1, 4)$, $(k, 2, 3)$ or $(1, 3, 3)$;
$C(a_1, a_2, a_3, a_4)$, where $(a_1, a_2, a_3, a_4) = (1, 2, k, l)$, $(2, 1, k, l)$, $(3, 1, 6, l)$, $(4, 1, 4, l)$, $(6, 1, 3, l)$, $(7, 1, 3, 8)$, $(8, 1, 3, 4)$, $(9, 1, 3, 3)$, $(13, 1, 3, 2)$, $(k, 1, 2, l)$, $(k, 2, 1, l)$, $(2^3, l)$, $(3, 2^3)$, $(1, 3, 2, l)$, $(2, 3, 1, l)$, $(3^2, 1, 12)$, $(4, 3, 1, 8)$, $(5, 3, 1, 7)$, $(9, 3, 1, 6)$, $(k, 3, 1, 5)$, $(1, 4, 1, l)$, $(k, 4, 1, 3)$, $(2, 4, 1, 4)$, $(1, 5, 1, 4)$, $(k, 6, 1, 2)$ or $(1, 7, 1, 2)$,

for any choice of integers $k, m \geq 1$ and $l \geq 2$.
Consequently, the new solutions $(3, 1, 6, l)$, $(4, 1, 4, l)$, $(6, 1, 3, l)$, $(7, 1, 3, 8)$, $(8, 1, 3, 4)$, $(9, 1, 3, 3)$ and $(13, 1, 3, 2)$ should be added to Table 1 (page 2207) with preserving the lexicographical ordering (the graph $(3, 1, 6, l)$ goes between 10^{th} and 11^{th}, etc).

10. Semigroups Working Group at Law’08, Janez Bernik. Semigroups of operators with nonnegative diagonals, 433 (2010), 2080–2087. Roman Drnovsek has written to say that the title of the working group was incorrectly typeset and should read: Semigroups Working Group at Law’08, Kranjska Gora. Kranjska Gora is a city in Slovenia where the working group originated the research in this paper. Janez Bernik was the corresponding author for this paper.

11. J. Li, W.C. Shiu, W.H. Chan, Some results on the Laplacian eigenvalues of unicyclic graphs, 430 (2009), 2080–2093. The authors have written to correct some mistakes in the proof of Theorem 3.3. In the proof of that theorem, some incorrect graphs at Figs. 7 and 8 were given. The proof of Theorem 3.3 is rewritten as follows. All notation and the numbering of figures, lemmas and theorems are the same as in the paper.
Theorem 3.3. For $n \geq 7$, $\mu_2(U) \geq 3$ for $U \in \mathcal{U}_n^+$; and the equality holds if and only if $U \cong U_n^i$ $(2 \leq i \leq \lfloor \frac{n+1}{2} \rfloor)$.
Proof. Let C_l be the unique cycle in U, $l \geq 3$.
If $l \neq 4$, then U contains $C_l + N_{n-l}$ as a spanning subgraph, where N_m is the null graph of order m. By Lemmas 2.1 and 3.1, we have $\mu_2(U) \geq \mu_2(C_l + N_{n-l}) \geq 3$.
If $l = 4$, then U contains one of the graphs $H_i + N_{n-6}$ $(i = 3, 4, 5)$ or $G_1 + N_{n-6}$ as a spanning subgraph, where H_i $(i = 3, 4, 5)$ are shown in Fig. 5 and G_1 is shown in Fig. 6. Since $\mu_2(H_3 + N_{n-6}) = \mu_2(H_4 + N_{n-6}) = \mu_2(H_5 + N_{n-6}) = 3$ and $\mu_2(G_1 + N_{n-6}) \doteq 3.414$. By Lemma 2.1, we have $\mu_2(U) \geq \min_{3 \leq i \leq 5} \{\mu_2(H_i + N_{n-6}),\ \mu_2(G_1 + N_{n-6})\} = 3$.
Hence we have $\mu_2(U) \geq 3$.
In the following, we shall show that for each $U \in \mathcal{U}_n^+$, $\mu_2(U) = 3$ if and only if $U \cong U_n^i$ for some i.

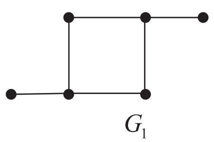

Fig. 6. Unicyclic graph G_1.

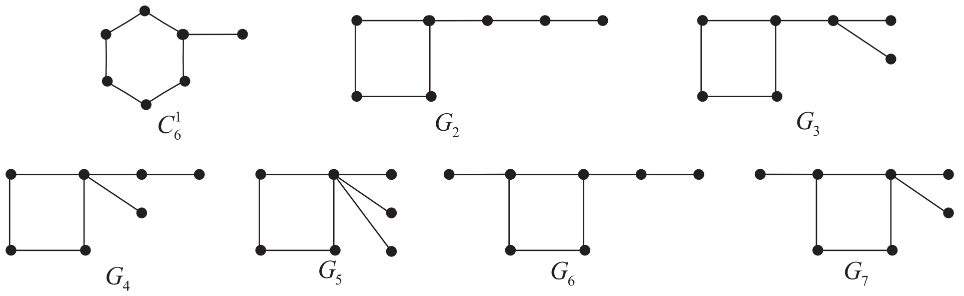

Fig. 7. Unicyclic graphs C_6^1 and G_i $(2 \leq i \leq 7)$.

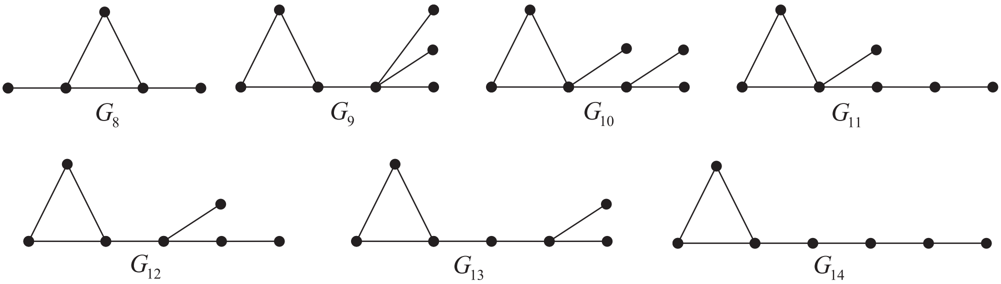

Fig. 8. Unicyclic graphs G_i $(8 \leq i \leq 14)$.

From Lemma 3.2, we know that $\mu_2(U_n^i) = 3$ for $n \geq 7$. Let C_l be the unique cycle in U. Then $C_l + N_{n-l}$ is a spanning subgraph of U, $n \geq 7$ and $n > l \geq 3$. By Lemma 2.1, $\mu_2(U) \geq \mu_2(C_l + N_{n-l}) = \mu_2(C_l)$. Since $\mu_2(U) = 3$, by Lemma 3.1 we have $l = 3, 4$ or 6.
Suppose $l = 6$. Since $n \geq 7$, $C_6^1 + N_{n-7}$ is a spanning subgraph of U, where C_6^1 is shown in Fig. 7. By Lemma 2.1 again, $3 = \mu_2(U) \geq \mu_2(C_6^1 + N_{n-7}) = \mu_2(C_6^1) \doteq 3.414 > 3$. It is impossible.
If $l = 4$, then U contains one of the graphs $G_1 + (n-6)K_1$ or $G_i + (n-7)K_1$ $(i = 2, 3, 4, 5, 6, 7)$ as a spanning subgraph, where G_1 is shown in Fig. 6 and G_i $(i = 2, 3, 4, 5, 6, 7)$ are shown in Fig.7. By Lemma 2.1, we have $\mu_2(U) \geq \min_{2 \leq i \leq 7}\{\mu_2(G_1 + (n-6)K_1),\ \mu_2(G_i + (n-7)K_1)\} = \mu_2(G_5 + (n-7)K_1) \doteq 3.058$. It is impossible too.
Suppose $l = 3$. If $U \ncong U_n^i$, then U contains one of the graphs $G_8 + (n-5)K_1$ or $G_i + (n-7)K_1$ $(i = 9, 10, 11, 12, 13, 14)$ as a spanning subgraph $(n \geq 7)$, where G_i $(i = 8, 9, 10, 11, 12, 13, 14)$ are shown in Fig. 8. By Lemma 2.1, we have $\mu_2(U) \geq \min_{9 \leq i \leq 14}\{\mu_2(G_8 + N_{n-5}), \mu_2(G_i + N_{n-7})\} = \mu_2(G_{11} + N_{n-7}) \doteq 3.117$. It is impossible. So by Lemma 3.2, for any $U \in \mathcal{U}_n^+$, if $\mu_2(U) = 3$, then $U \cong U_n^i$.
From the above discussions, the proof is completed.